جـــامـــعـــة مـحـمـد بـــن زايـــد
للـــذكـــاء الاصـــطـــنـــاعـــي
MOHAMED BIN ZAYED UNIVERSITY
OF ARTIFICIAL INTELLIGENCE

Core Courses Syllabi

MTH701 - Mathematical Foundation for AI

Title	Mathematical Foundation for AI
Code	MTH701
Loading	4 Credit-hours
Prerequisites	None
Catalog Description	This course provides a comprehensive mathematical foundation for artificial intelligence. It builds upon fundamental concepts in linear algebra, probability theory, and basic statistics and overviews basics and advanced topics that are frequently encountered in AI applications. The students will learn the basic mathematical concepts for main AI systems, as well as realistic applications in AI of mathematical tools.
Goal	This elective course aims to familiarize students with mathematical foundations of artificial intelligence.
Content	The course covers the following major modules: **(I)** Linear Algebra, **(II)** Probability Theory and **(III)** Basic Statistics
Recommended Textbooks	1. Golub, Gene H., and Charles F. Van Loan. *Matrix Computations*. Vol. 3. JHU press, 2012. 2. Evans, Michael J., and Jeffrey S. Rosenthal. *Probability and statistics: The science of uncertainty.* Macmillan, 2009. 3. Gilbert Strang, Introduction to Linear Algebra, 5th Edition, 2016
Recommended References & Supplemental Material	Relevant research papers, tech reports, and surveys for each topic, where needed, are identified in the teaching plan ahead. In addition, the following textbook may be useful: Trevor Hastie et al., *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, Second Edition, 2008

Teaching Week	Topics
1	**Matrix Multiplication** **Lecture** • Basic Algorithms and Notation • Structure and Efficiency • Block Matrices and Algorithms • Fast Matrix-Vector Products • Vectorization and Locality • Parallel Matrix Multiplication **Lab** • Instructor-led demonstration related to the topics taught in the week
2	**Matrix Analysis** **Lecture** • Basic Ideas from Linear Algebra • Vector Norms • Matrix Norms • The Singular Value Decomposition • Subspace Metrics • The Sensitivity of Square Systems • Finite Precision Matrix Computations **Lab** • Instructor-led demonstration related to the topics taught in the week
3	**General Linear Systems** **Lecture** • Triangular Systems • The Lower-Upper (LU) Factorization • Roundoff Error in Gaussian Elimination **Lab** • Instructor-led demonstration related to the topics taught in the week
4	**Special Linear Systems** **Lecture** • Diagonal Dominance and Symmetry • Positive Definite Systems • Banded Systems • Symmetric Indefinite Systems • Block Tridiagonal Systems • Vandermonde Systems • Classical Methods for Toeplitz Systems • Circulant and Discrete Poisson Systems **Lab** • Instructor-led demonstration related to the topics taught in the week

Teaching Week	Topics
5	**Orthogonalization and Least Squares** **Lecture** • The QR Factorization • The Full-Rank Least Squares Problem • Other Orthogonal Factorizations • The Rank-Deficient Least Squares Problem • Square and Underdetermined Systems **Lab** • Instructor-led demonstration related to the topics taught in the week
6	**Functions of Matrices** **Lecture** • Eigenvalue Methods • Approximation Methods • The Matrix Exponential • The Sign, Square Root, and Log of a Matrix **Lab** • Instructor-led demonstration related to the topics taught in the week
7	**Probability** **Lecture** • Probability Models • Properties of Probability Models • Uniform Probability on Finite Spaces • Conditional Probability and Independence **Lab** • Instructor-led demonstration related to the topics taught in the week
8	**Review** **Lecture and Tutorial** • Review and Exam Preparation
9	**Random Variables and Distributions** **Lecture** • Distributions of Random Variables • Discrete Distributions • Continuous Distributions • Cumulative Distribution Functions • Joint Distributions • Conditioning and Independence **Lab** • Instructor-led demonstration related to the topics taught in the week

Teaching Week	Topics
10	**Expectation** **Lecture** • The Discrete Case • The Absolutely Continuous Case • Variance • Covariance • Correlation • Conditional Expectation • Inequalities **Lab** • Instructor-led demonstration related to the topics taught in the week
11	**Sampling Distributions and Limits** **Lecture** • Sampling Distributions • Convergence in Probability • Monte Carlo Approximations • Normal Distribution Theory **Lab** • Instructor-led demonstration related to the topics taught in the week
12	**Statistical Inference and Likelihood Inference** **Lecture** • Inference Using a Probability Model, Statistical Models, Data Collection • The Likelihood Function • Inferences Based on the MLE • Distribution-Free Methods • Large Sample Behavior of the MLE (Advanced) **Lab** • Instructor-led demonstration related to the topics taught in the week

Teaching Week	Topics
13	**Bayesian Inference** **Lecture** • The Prior and Posterior Distributions • Inferences Based on the Posterior • Bayesian Computations • Choosing Priors **Lab** • Instructor-led demonstration related to the topics taught in the week
14	**Linear Regression and Correlation Analysis** **Lecture** • Least-Squares Method • Normal Regression Analysis **Lab** • Instructor-led demonstration related to the topics taught in the week
15	**Criteria for Evaluating the Goodness of Estimators** **Lecture** • The Unbiased Estimator • The Minimum Variance Unbiased Estimator • Sufficient Estimator • Consistent Estimator • Review and Exam Preparation **Lab** • Instructor-led demonstration related to the topics taught in the week