Core Courses Syllabi

ML703 - Probabilistic and Statistical Inference

Title	Probabilistic and Statistical Inference
Code	ML703
Loading	4 Credit-hours
Prerequisites	Familiarity with fundamental concepts in Probability, Linear Algebra, Statistics, and programming
Catalog Description	Probabilistic and statistical inference is the process of drawing useful conclusions about data populations or scientific truths from uncertain and noisy data. This course will cover the different modes of performing inference including statistical modelling, data-oriented strategies and explicit use of designs and randomization in analyses. Furthermore, it will provide an in-depth treatment to the broad theories (frequentists, Bayesian, likelihood) and numerous practical complexities (missing data, observed and unobserved confounding, biases) for performing inference. This course presents the fundamentals of statistical and probabilistic inference and shows how these fundamental concepts are applied in practice.
Goal	During this course, the students will develop an understanding of the broad field of probabilistic and statistical inference and use this information for making informed choices in analysing data. The goal of this course is to introduce basic concepts, motivate the students about the practical and scientific significance of reasoning about uncertainty and provide necessary background in frequentist and Bayesian approaches to statistical inference.
Content	Probabilistic learning, latent variable modelling, expectation maximization (EM), time series modelling, graphical models, Gaussian processes, intractable models, sampling methods, variational approximations, variational bayes, expectation propagation, belief propagation.
Recommended Textbooks	1. Koller D., and Friedman N., Probabilistic Graphical Models, Probabilistic Graphical Models: Principles and Techniques, MIT Press. ISBN: 9780262013192 2. Bishop, C., Pattern Recognition and Machine Learning. Berlin: Springer-Verlag. ISBN: 0387310738
Recommended References & Supplemental Material	The following textbooks may be useful: 1. Friedman, J., Hastie, T., & Tibshirani, R. (2001). The elements of statistical learning (Vol. 1, No. 10). New York: Springer series in statistics. ISBN 9780387848587 2. Wasserman, L. (2006). All of nonparametric statistics. Springer Science & Business Media. ISBN 9780387306230

Teaching Week	Topics
1	**Introduction to Probabilistic Learning** **Lecture** • Supervised, unsupervised learning from probabilistic perspective, basic rules of probability, • Bayesian learning (with examples) • Multivariate linear regression: maximum likelihood estimation, Posterior estimation • Multivariate gaussian models for regression **Lab** • Problem solving related to multivariate linear regression
2	**Modelling of the latent Variables** **Lecture** • Latent variables, Gaussians, multivariate Gaussians, • Probabilistic principal components analysis (PPCA), PPCA likelihood • Eigen decomposition of a covariance matrix, • Principal component analysis, PCA and eigenvectors, mutual information perspective **Lab** • Implementation of PPCA on different datasets for data visualization
3	**Modelling of the latent Variables** **Lecture** • Factor analysis, gradient methods for learning FA, • FA and PCA, canonical correlation analysis • Limitations of Gaussian, FA and PCA models • Mixture of Gaussians (MoG), clustering with a MoG • Maximum likelihood for a mixture model • The K-means algorithm as mixture of Gaussians **Lab** • Mathematical proofs and problem solving related to factor analysis
4	**Expectation Maximization** **Lecture** • Log likelihood, the expectation maximisation (EM) algorithm, the lower bound for EM • Expectation maximization for mixture of Gaussians • Expectation maximization for factor analysis • Expectation maximization for exponential family mixtures **Lab** • Implement the EM algorithm for mixture of Gaussians, factor analysis and exponential family of mixtures

Teaching Week	Topics
5	**Time Series Modelling** **Lecture** • Markov models, causal structure and latent variables, • Latent-chain models, state space models • Link between state space models and factor analysis • Linear-Gaussian state space models, linear dynamical systems • Kalman filtering, hidden markov models, link between HMMs and state space models, the HMM: forward–backward algorithm • Learning HMMs using EM, classification with HMMs **Lab** • Implement HMMs for speech analysis
6	**Graphical Models:** **Lecture** • Categories of graphical models, graphs, independence and factorisation • Factor graphs, factorisation and conditional independence • Undirected graphical models: Markov networks • Undirected graphs and factor graphs, their limitations • Directed acyclic graphical models, conditional independence in DAGs • Tree-structured graphical models, finding marginals in undirected trees, DAGs to factor graph • Back propagation on directed and A-directed graphs, learning and inference on graphical models **Lab** • Problem solving related to directed and undirected graphical models
7	**Model Selection, Hyperparameter Optimisation, and Gaussian Processes** **Lecture** • Learning model structure and complexity, • Overfitting and underfitting, Bayesian model selection, Bayesian Information Criterion (BIC), • Hyperparameters and evidence optimisation, • The Gaussian process • Regression with Gaussian process • Sampling from the Gaussian process **Lab** • Problem solving on complexity analysis of models • Experimental evaluations on overfitting, underfitting

Teaching Week	Topics
8	**Beyond Gaussian: Intractable Models** **Lecture** • Tractable and intractable models, non-linear and non-Gaussian models • Hierarchical models, noiseless ICA • Information maximization independent component analysis • Blind source separation, non-linear state space models • Boltzmann machine, learning in Boltzmann machine, restricted Boltzmann machines • Dynamic Bayesian networks, non-linear dimensional reduction • Multidimensional scaling, non-metric MDS, isomap, LLE **Lab** • Experiments evaluations on deep network parameter initialization using Restricted Boltzmann Machines (RBMs)
9	**Sampling Methods** **Lecture** • Monte-carlo integration, importance sampling, rejection sampling, sample size • Sample drawing methods, Markov chain monte carlo (MCMC) methods, Gibbs sampling • Contrastive divergence learning, practical MCMC, annealing • Hybrid/Hamiltonian monte carlo • Simulating the dynamical system • Langevin monte carlo, sequential monte carlo — particle filtering **Lab** • Implement different sampling methods • Problem solving related to MCMC
10	**Variational Approximations** **Lecture** • Expectations in statistical modelling • Distributed models • Free-energy-based variational approximation • KL divergence • Factored variational expectation maximization **Lab** • Problem solving and mathematical proofs related to variational approximation

Teaching Week	Topics
11	**Variational Bayes** **Lecture** • Mean-field factored HMM • Structured variational approximation, non-factored variational methods • Variational Bayes, conjugate-exponential models, conjugate-exponential variational Bayes • Augmented variational methods • Sparse GP approximations, variational sparse GP approximations **Lab** • Implement a variational Autoencoder to sample generation
12	**Expectation Propagation:** **Lecture** • State-space models, non-linear state space models • Extended Kalman filter (EKF) • Posterior optimization • Expectation propagation (EP) • Non-linear state space models expectation propagation message updates • EP for Gaussian processes, EP and alpha divergences **Lab** • Implement expectation propagation
13	**Belief Propagation I** **Lecture** • Belief propagation on directed and undirected graphs • Loopy belief propagation • Loopy BP as message-based expectation propagation • Loopy BP as tree-based reparameterization • Reparameterization on non-trees **Lab** • Mathematical proofs and problem solving related to belief propagation
14	**Belief Propagation II** **Lecture** • Loopy BP as tree-based reparameterization • Loopy BP and Bethe free energy • Bethe fixed point equations • Generalized belief propagation **Lab** • Instructor-led demonstration
15	**Review and Discussion**